Coding Isn't Programming

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What This Talk is About

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I've realized that some things that I've learned about writing concurrent programs apply to all programs.

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I have nothing new to say about that, and I'm tired of saying the same old stuff.

I've realized that some things that I've learned about writing concurrent programs apply to all programs.

So I'm going to talk about programming.

Algorithms

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I'm here because I wrote concurrent algorithms, not concurrent programs.

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An algorithm is not a program

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An algorithm is not a program, it's higher-level, more abstract.

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An algorithm is not a program, it's higher-level, more abstract.

It can be implemented in many programming languages.

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We don't have to write algorithms in a programming language.

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We shouldn't write algorithms in a programming language.

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We shouldn't write algorithms in a programming language.

Programming languages are complicated.

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Programming languages are complicated.

- They have to be efficiently executed.
- They have to handle large programs.

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Algorithms

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We shouldn't write algorithms in a programming language.

Programming languages are complicated.

Algorithms are neither executed nor large.

What language should we use?

~~What language should we use?~~

Don't get hung up on languages.

~~What language should we use?~~

Don't get hung up on languages.

Think about ideas, not the language they're expressed in.

Concurrent Programs

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Contain multiple threads of control that can be executed concurrently.

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Distributed programs are ones in which the threads are executed on different computers.

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- Actions of different threads can be interleaved in many ways.
- This implies a huge number of possible executions.
- Hard to think of all the ways they can go wrong.

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Debugging doesn't work.

- Can't be sure you've checked all relevant cases.
- A program can work fine, and a change that alters the relative execution rates of the threads can reveal a bug.
- Fixing one bug is likely to introduce a new bug.

Concurrent Programs

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Concurrent programs are very hard to get right.

Debugging doesn't work.

How to Write a Concurrent Program

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Figure out what part of what the program does involves concurrency.

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Figure out what part of what the program does involves concurrency.

The part that synchronizes the threads.

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Figure out what part of what the program does involves concurrency.

The part that synchronizes the threads.

Usually a small part of what the program does.

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Find a correct algorithm to do that part.

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- Maybe it's almost like one in a textbook.
- Maybe it's brand new.

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Find a correct algorithm to do that part.

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- Maybe it's brand new.
- Hard for the same reason concurrent programs are hard to get right.
- But algorithms are simpler than programs.

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- Programmers are good at coding.

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Figure out what part of what the program does involves concurrency.

Find a correct algorithm to do that part.

Implement the algorithm.

- That's coding.
- Lot's of languages and tools have been developed for coding.
- Programmers are good at coding.
- They're not so good at finding correct algorithms.

How to Write a Concurrent Program

Figure out what part of what the program does involves concurrency.

Find a correct algorithm to do that part.

Implement the algorithm.

Abstraction

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Something is usually called an *algorithm* only if it can be useful in many applications.

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Something is usually called an *algorithm* only if it can be useful in many applications.

The algorithm that describes how concurrency is implemented in a program is often useful only for that program.

I therefore call it an *abstract view*, or an *abstract program*, or simply an *abstraction*.

How to Write a Concurrent Program

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Find an abstract view of the program that describes how it handles concurrency.

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Find an abstract view of the program that describes how it handles concurrency.

A higher-level abstraction than the code.

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Thinking before you code.

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Thinking at a higher level than code

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It involves a new kind of thinking.

Programmers should learn how to do it.

How to Write a ~~Concurrent~~ Program

Find an abstract view of the program that describes how it handles concurrency.

Programmers are taught how to code, not how to abstract.

It involves a new kind of thinking.

Programmers should learn how to do it for all programs.

What's a Program?

What's a Program?

Any piece of code that requires thinking before coding.

What's a Program?

Any piece of code that requires thinking before coding.

Perhaps a complete program

What's a Program?

Any piece of code that requires thinking before coding.

- Perhaps a complete program or a method

What's a Program?

Any piece of code that requires thinking before coding.

Perhaps a complete program or a method or a complicated loop.

What's a Program?

Any piece of code that requires thinking before coding.

What's a Program?

Any piece of code that requires thinking before coding.

Any piece of code to be used by someone who doesn't want to read the code.

How to Write an Abstraction

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How to Write an Abstraction

Programs are written for many purposes.

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For most programs, you should write two things:

- What the program does.
- How the program does it.

What language should we use?

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If we want to build a tool to check that the *how* implies the *what*,

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If we want to build a tool to check that the *how* implies the *what*, then we need a precise language.

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If we want to build a tool to check that the *how* implies the *what*, then we need a precise language.

Needed to ensure that a concurrent program does what it should.

~~What language should we use?~~

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If we want to build a tool to check that the *how* implies the *what*, then we need a precise language.

Needed to ensure that a concurrent program does what it should.

I designed TLA$^+$ for that.

~~What language should we use?~~

Don't get hung up on languages.

If we want to build a tool to check that the *how* implies the *what*, then we need a precise language.

Needed to ensure that a concurrent program does what it should.

Not needed for most programs.

A Trivial Example

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Write a function to compute the largest element in an array of 32-bit integers.

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What: Return the largest element in an integer array A.

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What: Return the largest element in an integer array A.

How: Examine $A[0]$, $A[1]$, ... in turn, letting x be the largest value found, and return x.

Here's the code, written in Rust

```
fn ArrayMax(A: &[i32]) -> i32 {
    let mut x = A[0];
    let mut i = 1;
    while i < A.len() {
        if A[i] > x {
            x = A[i];}
        i += 1;  }
    x }
```

Here's the code, written in Rust (which I don't know)

```
fn ArrayMax(A: &[i32]) -> i32 {
    let mut x = A[0];
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Raise your hand if you see the bug.

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It's not a coding bug.

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        i += 1;  }
    x }
```

Raise your hand if you see the bug.

It's not a coding bug. It's a bug in the *What*.

What: Return the largest element in an integer array A.

What: Return the largest element in an integer array A.

If A has no element, then there is no largest one.

What: Return the largest element in an integer array A.

If A has no element, then there is no largest one.

There's an obvious fix to the *What*

What: Return the largest element in an integer array A,
or an error value if A has no element.

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What: Return the largest element in an integer array A, or an error value if A has no element.

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There's an obvious fix to the *What* and to the *How*:

What: Return the largest element in an integer array A, or an error value if A has no element.

If A has no element, then there is no largest one.

There's an obvious fix to the *What* and to the *How*:

```
fn ArrayMax(A: &[i32]) -> Result<i32, &'static str> {
    if A.is_empty() {
        return Err("EmptyArray"); }
    let mut x = A[0];
    let mut i = 1;
    while i < A.len() {
        if A[i] > x {
            x = A[i];}
        i += 1;  }
    Ok(x) }
```

Abstraction

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Let's now see how abstraction removes coding details and helps us understand why the program does the right thing.

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Remember, we wouldn't really do this on such a tiny example.

Abstraction

Let's now see how abstraction removes coding details and helps us understand why the program does the right thing.

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But it will illustrate how abstraction works.

Abstracting the What

What: Return the largest element in an integer array A or an error value if A has no element.

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The function call/return is a coding detail.

Abstracting the What

What: Return the largest element in an integer array A or an error value if A has no element.

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We're interested in how the largest element of A is found.

Abstracting the What

What: Set x to the largest element in an integer array A or an error value if A has no element.

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Abstracting the What

What: Set x to the largest element in an integer array A or an error value if A has no element.

Why just integers?

Abstracting the What

What: Set x to the largest element in an array A of numbers or an error value if A has no element.

Why just integers?

Abstracting the What

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Why an array?

Abstracting the What

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Why an array?

I assume we're interested in the elements, not how they're arranged.

Abstracting the What

What: Set x to the largest element in a multiset A of numbers or an error value if A has no element.

Why an array?

I assume we're interested in the elements, not how they're arranged.

Abstracting the What

What: Set x to the largest element in a multiset A of numbers or an error value if A has no element.

Why an array?

I assume we're interested in the elements, not how they're arranged.

It's a multiset, not a set, because it can contain multiple copies of the same element.

What: Set x to the largest element in a multiset A of numbers or an error value if A has no element.

What: Set x to the largest element in a multiset A of numbers or an error value if A has no element.

I don't like this part because it complicates the code.

What: Set x to the largest element in a multiset A of numbers
or an error value if A has no element.

I don't like this part because it complicates the code.

Can we find a different *What* that's just as good
but has a simpler implementation?

Of course this is silly for such a simple example.

Of course this is silly for such a simple example.
The complication is about two lines of code.

Of course this is silly for such a simple example.

But when I wrote a program, often I would:

Of course this is silly for such a simple example.

But when I wrote a program, often I would:

– Decide what I wanted the program to do.

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But when I wrote a program, often I would:

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– Realize coding it would be a lot of work.

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But when I wrote a program, often I would:

- Decide what I wanted the program to do.
- Realize coding it would be a lot of work.
- Figure out what I really needed it to do.

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But when I wrote a program, often I would:

- Decide what I wanted the program to do.
- Realize coding it would be a lot of work.
- Figure out what I really needed it to do.

The result was a *What* that might not have everything I wanted

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But when I wrote a program, often I would:

- Decide what I wanted the program to do.
- Realize coding it would be a lot of work.
- Figure out what I really needed it to do.

The result was a *What* that might not have everything I wanted, but had everything I needed.

This took time

Of course this is silly for such a simple example.

But when I wrote a program, often I would:

– Decide what I wanted the program to do.

– Realize coding it would be a lot of work.

– Figure out what I really needed it to do.

The result was a *What* that might not have everything I wanted, but had everything I needed.

This took time, but it saved much more time writing the code.

What: Set x to the largest element in a multiset A of numbers or an error value if A has no element.

What: Set x to the largest element in a multiset A of numbers ~~or an error value if A has no element.~~

We want to eliminate this

What: Set x to the largest element in a multiset A of numbers

We want to eliminate this, which requires modifying this.

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How to do that is not obvious to most programmers.

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It's obvious to me because I was educated as a mathematician

What: Set x to the largest element in a multiset A of numbers.

We want to eliminate this, which requires modifying this.

How to do that is not obvious to most programmers.

It's obvious to me because I was educated as a mathematician, so I know what the largest element of an empty set should equal.

Set x to the smallest number $\geq$ all elements of A.

What: ~~Set x to the largest element in a multiset A of numbers.~~

We want to eliminate this, which requires modifying this.

Here's how.

Set x to the smallest number $\geq$ all elements of A.

What: ~~Set x to the largest element in a multiset A of numbers.~~

We want to eliminate this, which requires modifying this.

Here's how.

The two are equivalent if A is not empty.

Set x to the smallest number $\geq$ all elements of A.

What: ~~Set x to the largest element in a multiset A of numbers.~~

We want to eliminate this, which requires modifying this.

Here's how.

The two are equivalent if A is not empty.

But what if A is empty?

What: Set x to the smallest number $\geq$ all elements of A.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

Why is this true?

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

I'm richer than everyone living in Bodie, CA.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

I'm richer than everyone living in Bodie, CA.

I'm poorer than everyone living in Bodie, CA.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

I'm richer than everyone living in Bodie, CA.

I'm poorer than everyone living in Bodie, CA.

No one lives in Bodie, CA.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

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This is simple (mathematical) logic.

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No one lives in Bodie, CA.

This is simple (mathematical) logic.

To think rationally, you have to understand simple logic.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

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I'm poorer than everyone living in Bodie, CA.

No one lives in Bodie, CA.

This is simple (mathematical) logic.

To think rationally, you have to understand simple logic.

Programmers should think rationally.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

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Mathematicians (sometimes) define $-\infty$ to be the smallest number.

What: Set x to the smallest number $\geq$ all elements of A.

If A is empty, then every number is $\geq$ all elements of A.

If A is empty, then x is set to the smallest number.

Mathematicians (sometimes) define $-\infty$ to be the smallest number.

So this *What* implies that if A is empty, then x is set to the number $-\infty$.

The *How*

The *How*

let $B = A$ **and** $x = -\infty$ **;**

The *How*

let $B = A$ **and** $x = -\infty$;
while B not empty {

}

The *How*

let $B = A$ **and** $x = -\infty$;
while B not empty {
    let $i =$ any element of B ;

}

The *How*

let $B = A$ **and** $x = -\infty$;
while B not empty {
  let $i =$ any element of B ;

}

This is nondeterministic; there are many possible executions.

The *How*

let $B = A$ **and** $x = -\infty$;
while B not empty {
  let $i =$ any element of B ;
  let $B = B$ with i removed ;
}

The *How*

```
let  B = A  and  x = −∞ ;
while  B not empty  {
      let  i = any element of B ;
      let  B = B with i removed ;
      if  i > x  { let x = i }        }
```

The *How*

let $B = A$ **and** $x = -\infty$;
while B not empty {
    let $i =$ any element of B ;
    let $B = B$ with i removed ;
    if $i > x$ { **let** $x = i$ } }

I used an *ad hoc* combination of programming-language notation

The *How*

```
let B = A and x = −∞ ;
while B not empty {
    let i = any element of B ;
    let B = B with i removed ;
    if i > x { let x = i }        }
```

I used an *ad hoc* combination of programming-language notation

The *How*

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How can we convince someone that this implements the *What*?

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I believe most programmers could only show that some executions compute the right value of x.

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But we must show **every** execution computes the right value of x.

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How can we convince someone that this implements the *What*?

I believe most programmers could only show that some executions compute the right value of x.

But we must show **every** execution computes the right value of x.

Seeing how to do this requires thinking about executions.

What is an Execution?

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An obvious answer:

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An obvious answer:

A sequence of steps.

What is an Execution?

An obvious answer:

A sequence of steps.

Each step is the execution of a part of the code.

What is an Execution?

A usually better answer:

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A usually better answer:

No rule applies to all programs.

What is an Execution?

A usually better answer:

A sequence of states.

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A step is a pair of consecutive states

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We suppose A has two copies of 2 and one copy of 3.

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Let's write this multiset as $\{\{2, 2, 3\}\}$ or $\{\{2, 3, 2\}\}$ or $\{\{3, 2, 2\}\}$.

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A usually better answer:

A sequence of states.

A step is a pair of consecutive states that describes execution of a part of the code.

To see how we describe an execution this way, we look at one possible execution.

We suppose A has two copies of 2 and one copy of 3.

Let's write this multiset as $\{\{2, 2, 3\}\}$ or $\{\{2, 3, 2\}\}$ or $\{\{3, 2, 2\}\}$.

I'll write it this way

```
let  B = {{2, 3, 2}}  and  x = −∞ ;
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A state will describe the values of B and x and perhaps other stuff.

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$$\begin{bmatrix} B = & \\ x = & \end{bmatrix}$$

A state will describe the values of B and x and perhaps other stuff.

I'll just show the values of B and x.

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let B = {{2,3,2}} and x = −∞ ;
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What are their initial values?

let $B = \{\{2, 3, 2\}\}$ **and** $x = -\infty$;
while B not empty {
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What are their initial values?

An obvious answer: some standard initial value we'll call "?" .

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$$\begin{bmatrix} B = ? \\ x = ? \end{bmatrix}$$

We first execute this statement.

let $B = \{\{2,3,2\}\}$ **and** $x = -\infty$ **;**
while B not empty {
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To understand this abstract program

let $B = \{\{2, 3, 2\}\}$ **and** $x = -\infty$;
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To understand this abstract program, we want an execution to be as simple as possible.

let $B = \{\{2,3,2\}\}$ **and** $x = -\infty$;
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To understand this abstract program, we want an execution to be as simple as possible.

This means making an execution have as few steps as possible.

let $B = \{\{2,3,2\}\}$ **and** $x = -\infty$;
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These two steps are of no interest.

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So we let this statement define the initial state.

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And we can rewrite it like this.

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What's the next state?

Code doesn't say what constitutes a step.

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To keep the abstraction simple

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To keep the abstraction simple, I will make evaluating the **while** loop's test

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To keep the abstraction simple, I will make evaluating the **while** loop's test and evaluating its body one step.

I won't bother defining a way to make the pseudocode say that's all one step.

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Executing the next step can let i equal 2 or 3.

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The next two steps have to choose 2.

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The next step finds B empty, so it exits the loop without changing B or x.

That's an uninteresting step.

So we declare that the execution terminates when B is empty.

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I don't know how to say that with code.

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(It's easy to say in TLA$^+$.)

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But don't get hung up on languages.

Understand what the executions are.

Don't worry about how they're described.

What's a State?

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For executions to be a useful way to think about a program:

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We can choose states this way for abstractions that describe actual programs because:

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Programs are executed on computers.

What's a State?

For executions to be a useful way to think about a program:

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We can choose states this way for abstractions that describe actual programs because:

Programs are executed on computers.

What a computer does next depends only on its current state

What's a State?

For executions to be a useful way to think about a program:

The possible next states must depend only on the current state, not on any previous state.

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Something true of every state of every execution is called an *invariant* of the program.

You don't understand why a program does the right thing

You don't understand why a program does the right thing,

for example terminating with the right answer

You don't understand why a program does the right thing,
unless you know the invariant that ensures it does the right thing.

Here is the invariant for our example

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And to show that the invariant implies correctness, we show:

3. It implies $x = max(A)$ in a terminated state.

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Can you figure out what values those are?

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Here's an example of how TLA^+ works in practice.

DOI:10.1145/2699417

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S3 is just one of many AWS services that store and process data our customers have entrusted to us. To safeguard that data, the core of each service relies on fault-tolerant distributed algorithms for replication, consistency, concurrency control, auto-scaling, load balancing, and other coordination tasks. There are many such algorithms in the literature, but combining them into a cohesive system is a challenge, as the algorithms must usually be modified to interact properly in a real-world system. In addition, we have found it necessary to invent algorithms of our own. We work hard to avoid unnecessary complexity, but the essential complexity of the task remains high.

Complexity increases the probability of human error in design, code, and operations. Errors in the core of the system could cause loss or corruption of data, or violate other interface contracts on which our customers depend. So, before launching a service, we need to reach extremely high confidence that the core of the system is correct. We have found the standard verification techniques in industry are necessary but not sufficient. We routinely use deep design reviews, code reviews, static code analysis, stress testing, and fault-injection testing but still find that subtle bugs can hide in complex concurrent fault-tolerant systems. One reason they do is that human intuition is poor at estimating the true probability of supposedly "extremely rare" combinations of events in systems operating at a scale of millions of requests per second.

» key insights

- **Formal methods find bugs in system designs that cannot be found through any other technique we know of.**
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BY CHRIS NEWCOMBE, TIM RATH, FAN ZHANG, BOGDAN MUNTEANU, MARC BROOKER, AND MICHAEL DEARDEUFF

How Amazon Web Services Uses Formal Methods

SINCE 2011, ENGINEERS at Amazon Web Services (AWS) have used formal specification and model checking to help solve difficult design problems in critical systems. Here, we describe our motivation and experience, what has worked well in our problem domain, and what has not. When discussing personal experience we refer to the authors by their initials.

At AWS we strive to build services that are simple for customers to use. External simplicity is built on a hidden substrate of complex distributed systems. Such complex internals are required to achieve high availability while running on cost-efficient infrastructure and cope with relentless business growth. As an example of this growth, in 2006, AWS launched S3, its Simple Storage Service. In the following six years, S3 grew to store one

S3 is just one of many AWS services that store and process data our customers have entrusted to us. To safeguard that data, the core of each service relies on fault-tolerant distributed algorithms for replication, consistency, concurrency control, auto-scaling, load balancing, and other coordination tasks. There are many such algorithms in the literature, but combining them into a cohesive system is a challenge, as the algorithms must usually be modified to interact properly in a real-world system. In addition, we have found it necessary to invent algorithms of our own. We work hard to avoid unnecessary complexity, but the essential complexity of the task remains high.

Complexity increases the probability of human error in design, code, and operations. Errors in the core of the system could cause loss or corruption of data, or violate other interface contracts on which our customers depend. So, before launching a service, we need to reach extremely high confidence that the core of the system is correct. We have found the standard verification techniques in industry are necessary but not sufficient. We routinely use deep design reviews, code reviews, static code analysis, stress testing, and fault-injection testing but still find that subtle bugs can hide in complex concurrent fault-tolerant systems. One reason they do is that human intuition is poor at estimating the true probability of supposedly "extremely rare" combinations of events in systems operating at a scale of millions of requests per second.

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These are fundamental design flaws

TLA^+ finds

- ~~Formal methods find~~ bugs in system designs that cannot be found through any other technique we know of.

These are fundamental design flaws, not just simple coding errors.

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These are fundamental design flaws.

Very expensive to fix after the code is written

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Very expensive to fix after the code is written because it requires extensive recoding.

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■ ~~Formal methods find~~ bugs in system designs that **cannot be found** through any other technique we know of.

These are fundamental design flaws.

Very expensive to fix after the code is written because it requires extensive recoding.

And often not found until the code has been released to users.

TLA$^+$ finds

- ~~Formal methods find~~ bugs in system designs that cannot be found through any other technique we know of.

These are fundamental design flaws.

Very expensive to fix after the code is written because it requires extensive recoding.

But Amazon engineers find these flaws before any code is written.

Is Abstraction Useful Just for Concurrency?

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The code for handling concurrency is important, but it's small.

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What about the rest of the program?

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What about the rest of the program?

I know of just one case in which an entire system system was built starting with a TLA$^+$ abstraction.

Rosetta

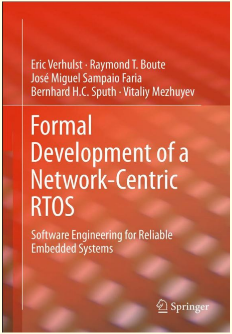
Rosetta

European Space Agency spacecraft that explored a comet.

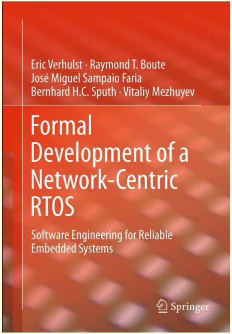
Rosetta

European Space Agency spacecraft that explored a comet.

Several of its instruments were controlled by
the Virtuoso real-time operating system.



The next version of Virtuoso.



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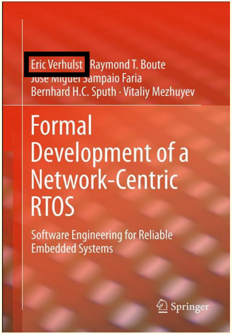

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Here's an email from Eric Verhulst,
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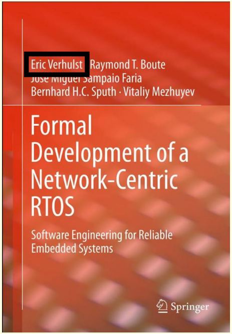

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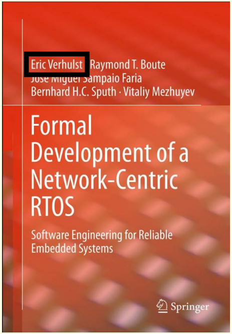

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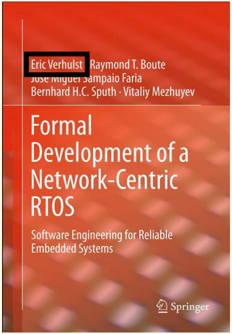

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You do it with a ~~cleaner architecture~~ better high-level design, which comes from thinking about an abstraction, not about the code.

It doesn't come from thinking in a programming language.

Sometimes what a program should do can't be stated precisely.

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Here's an example I encountered.

TLATEX — the TLA$^+$ pretty-printer

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The input:

```
Foo => /\ a   = b
       /\ ccc = d
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TLATEX — the TLA$^+$ pretty-printer

The input:

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The right output:

$$\wedge\ aaa + bb = c$$
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The user probably didn't wanted these aligned.

There is no precise definition of correct alignment.

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We can't describe precisely what the user wants.

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Wrong.

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The program has to do something.

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Wrong.

The program has to do something.

Not knowing precisely what it *should* do means
we have to think abstractly about what it *will* do.

It's impossible to specify the best pretty-printer.

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But the program has to do something.

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But the program has to do something.

Writing stream-of-consciousness code doesn't produce a good program.

My Abstraction

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6 rules plus definitions (in comments).

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Example:

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This is defined precisely (mostly in English).

Why Did I Write This Abstraction?

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I did a lot of debugging of the rules, aided by debugging code to report what rules were being used.

The few bugs in implementing the rules were easy to catch.

Had I just written code, it would have taken me much longer and not produced formatting as good.

What is Typical About This Abstraction

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You write an abstraction to help you think about the problem before you think about the code.

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No method is best for all programs.

Thinking is always better than not thinking before coding.

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Some people say that you shouldn't think too much before coding.

Thinking is always better than not thinking before coding.

Some people say that you shouldn't think too much before coding.

I say that too little thinking is a much more serious and much more common problem than too much thinking.

How to Think

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> "Writing is nature's way of letting you know how sloppy your thinking is."
>
> *Guindon*

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Writing helps you think better.
Thinking better helps you write better.

It’s a virtuous cycle.

To begin, most people must learn to write better.

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This means writing to convince others.

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It's too easy to convince yourself of something that's not true.

To begin, most people must learn to write better.

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You have to learn to read what you wrote the way others might read it.

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This means writing to convince others.

You have to learn to read what you wrote the way others might read it.

Perhaps other readers can teach you that.

How to Think Abstractly

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Abstraction is what I'm good at.

How to Think Abstractly

Abstraction is what I'm good at.

And being good at it is why I was invited to speak to you.

How to Think Abstractly

Abstraction is what I'm good at.

How did I become good at it?

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Abstraction is what I'm good at.

How did I become good at it?

In large part by being educated as a mathematician.

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Math abstracts from two sheep and two goats to the number 2.

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I don't know how you should learn to be better at abstraction.

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Because it's mathematics, TLA$^+$ teaches some users,
but it may be too hard for most programmers.

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Try asking them.

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If the program is simple, very little writing is necessary.
But it takes thinking to know if it's simple.

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For non-simple programs, abstract thinking (above the code level) can avoid errors and lead to better, easier to write code.

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Programming should be thinking followed by coding.

Thinking requires writing.

If the program is simple, very little writing is necessary.

For non-simple programs, abstract thinking (above the code level) can avoid errors and lead to better, easier to write code.

A non-simple program can be anything from a complete system to a complicated loop.

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In this abstraction, a program does the right thing because it satisfies an invariant.

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A state should contain all the information that can affect what future states are possible.

In this abstraction, a program does the right thing because it satisfies an invariant.

Understanding the program requires understanding that invariant.

Things to Remember

Don't get hung up on languages.

Things to Remember

Don't get hung up on languages.

Especially not on programming languages.

A Postscript

Why Programs Should Have Bugs

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I started programming in 1957.

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We can write much more complex programs now.

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Part of the reason is better programming languages.

Why Programs Should Have Bugs

I started programming in 1957.

We can write much more complex programs now.

Part of the reason is better programming languages.

But the major reason is that we have libraries of programs our programs can use.

The hardest part of programming is now figuring out how to use those library programs,

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A program can have bugs only if there is a precise description of what it should do.

Part of that description is language dependent.

- That part is usually simple.
- It's often implied by how the program is called, especially for strongly-typed languages.

The hardest part of programming is now figuring out how to use those library programs, because many of them can never have bugs.

A program can have bugs only if there is a precise description of what it should do.

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But the most useful programs do more complex things.

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A program can have bugs only if there is a precise description of what it should do.

Part of that description is language dependent.

But the most useful programs do more complex things.

You shouldn't have to read the code to understand those things.

They should have an abstract, language-independent description.

Few programs have such a description.

Few programs have such a description.

Many have no description at all.

Few programs have such a description.

I won't bother giving you horror stories of how this has made it difficult or impossible for me to use some library programs.

Few programs have such a description.

I won't bother giving you horror stories of how this has made it difficult or impossible for me to use some library programs.

Instead, I'll tell you about an organization that did a pretty good job of providing precise descriptions.

The W3C JavaScript Standard

W3C = World Wide Web Consortium

The W3C JavaScript Standard

There has been a lot of work on verifying that what a program should do is implied by how it does it.

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There has been a lot of work on verifying that what a program should do is implied by how it does it.

Initiated by a 1967 paper of Robert Floyd.

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This view works fine for sequential programs, and I expect most JavaScript programs are sequential.

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with web pages for viewing it.

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The code controlling the video is executed by one thread.

The code handling mouse clicks is executed by a diferent thread.

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It required a lot of debugging.

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We should be able to do better than that.

“That’s all folks!”